Interpolate Points on a Shape

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1 UNIFORMLY SPACED INTERPOLATED POINTS

We assume we have to vectors, $\mathbf{x}$ and $\mathbf{y}$, that store the N points $\{x_i, y_i\}_{i=1}^{N}$ defining a shape. We want to find n evenly spaced points between every point defining the shape. Consider the line segment connecting the points (x_i, y_i) and (x_{i+1}, y_{i+1}), as shown in Fig. 1. The length, ℓ, of this line segment (i.e. the distance between the two points) is

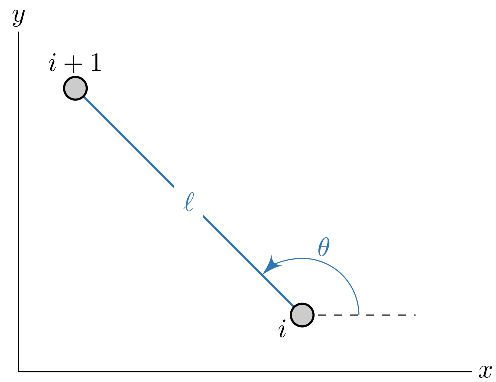

Figure 1: Line segment connecting points i and $i + 1$.

$$\boxed{\ell = \sqrt{\left(x_{i+1} - x_i\right)^2 + \left(y_{i+1} - y_i\right)^2}} \tag{1}$$

The angle (measured counter-clockwise) that the line segment forms with the horizontal line $y = y_i$ can be found using the four-quadrant inverse tangent.

$$\tan\theta = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \quad \rightarrow \quad \boxed{\theta = \text{arctan2}\left(y_{i+1} - y_i, x_{i+1} - x_i\right)} \tag{2}$$

We want to interpolate n uniformly spaced points between points i and $i + 1$, which means that the distance between each interpolated point is then ℓ/n. It follows that the first interpolated point is a distance ℓ/n from point i, the second interpolated point is a distance $2\ell/n$ from point i, and so on. Thus, the j^{th} interpolated point is a distance $j\ell/n$ from point i. Since all the interpolated points lie on the line segment connecting points i and $i+1$, they all make the same angle θ with the horizontal line emanating from point i (see Fig. 1). Thus, the x- and y- coordinates of the j^{th} interpolated point can be calculated as

$$\boxed{x_{ji} = x_i + \frac{j\ell\cos\theta}{n}} \tag{3}$$

$$\boxed{y_{ji} = y_i + \frac{j\ell\sin\theta}{n}} \tag{4}$$